

# Maxima vs MuPAD

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## Introduction

The purpose of this document is to compare two computer algebra systems **Maxima** and **MuPAD** with the help of 131 mainly mathematical problems proposed by M. WESTER 1994; see <http://math.unm.edu/wester/cas/Paper.ps>.

The computer algebra system **Maxima** is based on DOE **Macsyma** but it does not (yet) have all functionality of **Macsyma**. It is distributed under the GNU Public License and is freely available from <http://maxima.sourceforge.net>. In his publication mentioned above M. WESTER used **Macsyma** version 419.0. **Maxima** version used here is 5.9.0, which is said to be "a development version of **Maxima**".

The computer algebra system **MuPAD** has been developed at the University of Paderborn, Germany. It is a commercial product, see <http://www.mupad.com>, but there are some free versions available "for members of non-profit educational and scientific institutions". In his publication mentioned above M. WESTER used **MuPAD** version 1.2.1a. Later (1995) P. ZIMMERMANN used version 1.2.2 to solve M. WESTER's test problems, see <http://krum.rz.uni-mannheim.de/cabench/mupadwest.ps>. The **Darwin** version of **MuPAD** used here is 2.5.0.

Apart from the coloring and scoring system the structure of the document and the formulation of the problems follow closely the papers of M. WESTER and P. ZIMMERMANN mentioned above. Also in preparing the solutions of the problems the corresponding solutions presented by M. WESTER [**Macsyma**] and P. ZIMMERMANN [**MuPAD**] have been very useful, which is gratefully acknowledged. Every problem and its solution is presented using the following scheme:

1. Problem n. [The statement of the problem.]
2. **Maxima:**
3. (Cn) [Maxima input expression(s).]
4. (Dn) [The output(s) of Maxima input expression(s) centered.]
5. **MuPAD:**

6. `>> [MuPAD input expression(s).]`
7. `[Blank line.]`
8. `[The output(s) of MuPAD input expression(s) centered.]`

The color of the keyword Maxima/MuPAD reflects the completeness of the solution of the problem in the following way:

**Maxima/MuPAD** The problem is solved completely and easily.

**Maxima/MuPAD** The problem is solved almost completely and/or quite easily.

**Maxima/MuPAD** The problem is solved only partially and/or very uneasily.

**Maxima/MuPAD** The problem is not solved.

**Maxima/MuPAD** The solution is wrong.

After the sections presenting the problems and their solutions there is a summary containing the results in tables, which also include the times of the calculations. Section Colophon explains the methods and lists the programs used in preparation of this document.

## 1 Numbers

Problem 1. Compute 50!.

**Maxima:**

```
(C1) 50!;  
(D1)  
  
3041409320171337804361260816606476884437764156896051200000000000
```

**MuPAD:**

```
>> 50!  
  
3041409320171337804361260816606476884437764156896051200000000000
```

Problem 2. Find the prime factorization of 50!.

**Maxima:**

```
(C2) factor( 50! );  
      47 22 12 8 4 3 2 2 2  
(D2) 2 3 5 7 11 13 17 19 23 29 31 37 41 43 47
```



Maxima:

```
(C6) float(1/7);  
(D6) .1428571428571428
```

MuPAD:

```
>> numlib::decimal(1/7)  
  
0, [1, 4, 2, 8, 5, 7]
```

Problem 7. Represent  $\pi$  as a continued fraction.

Maxima:

```
(C7) cfdisrep(cf(float(%pi)));  
(D7) 3 + -----  
      1  
      7 + -----  
          1  
          15 + -----  
              1  
              1 + ---  
                  292
```

MuPAD:

```
>> contfrac(PI, 5)  
  
3 + -----  
    1  
    7 + -----  
        1  
        15 + -----  
            1  
            1 + ...
```

Problem 8. Simplify  $\sqrt{2\sqrt{3}+4}$ .

Maxima:

```
(C8) load(sqdnst)$ sqrtddenest( sqrt(2*sqrt(3) + 4) );  
(C9)  
(D9) Sqrt(3) + 1
```

MuPAD:

```
>> simplify( sqrt(2*sqrt(3) + 4), sqrt )
```

$$\sqrt[3]{\frac{1}{2}} + 1$$

Problem 9. Simplify  $\sqrt{14 + 3\sqrt{3 + 2\sqrt{5 - 12\sqrt{3 - 2\sqrt{2}}}}}$ .

Maxima:

```
(C9) load(squdnst)$
(C10) sqrt(denest(scanmap(lambda([u], radcan(sqrt(denest(u))),
sqrt(14+3*sqrt(3+2*sqrt(5-12*sqrt(3-2*sqrt(2)))))) ) ));
(D10) Sqrt(2) + 3
```

MuPAD:

```
>> simplify(
&> sqrt(14+3*sqrt(3+2*sqrt(5-12*sqrt(3-2*sqrt(2))))), sqrt )
```

$$\sqrt[2]{\frac{1}{2}} + 3$$

Problem 10. Simplify  $2\infty - 3$ .

Maxima:

```
(C10) limit( 2*inf - 3 );
(D10) INF
```

MuPAD:

```
>> 2*infinity - 3
infinity
```

## 2 Statistics

Problem 11. Compute the standard deviation of the sample  $\{1, 2, 3, 4, 5\}$ .

Maxima:

```
(C11) mean(x) := apply("+", x)/length(x)$
(C12) stdev(x) := sqrt(apply("+", (x - mean(x))^2)/(length(x)-1))$
(C13) stdev( [1, 2, 3, 4, 5] );
```

$$\frac{\sqrt{5}}{\sqrt{2}}$$

MuPAD:

```
>> stats::stdev( [1, 2, 3, 4, 5], Sample )
```

$$\frac{\frac{1}{2} \sqrt{5}}{2}$$

Problem 12. An experiment was repeated 10 times giving the results {1, -2, 3, -4, 5, -6, 7, -8, 9, 10}. Using *t*-distribution calculate the probability of true mean being greater than 5.

Maxima:

MuPAD:

```
>> sample := [1,-2,3,-4,5,-6,7,-8,9,10]: stats::tTest( sample, 5 )
```

$$\frac{21 \sqrt{2} + 10 \sqrt{10} + 29 \sqrt{29}}{290}, 0.05756660092$$

Problem 13. Do the previous problem using normal distribution.

Maxima:

MuPAD:

```
>> sample := [1, -2, 3, -4, 5, -6, 7, -8, 9, 10]:
>> float(stats::tTest( sample, 5, Normal ))
```

$$[-1.743955077, 0.04058346175]$$

### 3 Algebra

Problem 14. Compute the normal form of the rational expression  $(x^2 - 4)/(x^2 + 4x + 4)$ .

Maxima:

```
(C14) factor( (x^2 - 4)/(x^2 + 4*x + 4) );
(D14)          x - 2
          -----
          x + 2
```

MuPAD:

```
>> normal( (x^2 - 4)/(x^2 + 4*x + 4) )
          x - 2
          -----
          x + 2
```

Problem 15. Simplify  $(e^x - 1)/(e^{x/2} + 1)$ .

Maxima:

```
(C15) radcan( (exp(x) - 1)/(exp(x/2) + 1) );
(D15)          x/2
          %E   - 1
```

MuPAD:

```
>> simplify( (exp(x) - 1)/(exp(x/2) + 1) )
          / x \
          exp| - | - 1
          \ 2 /
```

Problem 16. Expand the expression  $(x + 1)^{20}$ . Differentiate the result and factorize the derivative.

Maxima:

```
(C16) factor(diff(expand( (x + 1)^20 ), x));
(D16)          20 (x + 1)19
```

MuPAD:

```
>> factor(diff(expand( (x + 1)^20 ), x))
          20 (x + 1)19
```

Problem 17. Factor  $x^{100} - 1$ .

Maxima:

```
(C17) factor( x^100 - 1 );
(D17) (x - 1) (x + 1) (x^2 + 1) (x^4 - x^3 + x^2 - x + 1)
      (x^4 + x^3 + x^2 + x + 1) (x^8 - x^6 + x^4 - x^2 + 1)
      (x^20 - x^15 + x^10 - x^5 + 1) (x^20 + x^15 + x^10 + x^5 + 1)
      (x^40 - x^30 + x^20 - x^10 + 1)
```

MuPAD:

```
>> factor( x^100 - 1 )
(x - 1) (x + 1) (x^2 + 1) (x^2 + x^3 + x^4 + 1)
      (- x^2 + x^3 - x^4 + 1) (- x^2 + x^4 - x^6 + x^8 + 1)
      (x^5 + x^10 + x^15 + x^20 + 1) (- x^5 + x^10 - x^15 + x^20 + 1)
      (- x^10 + x^20 - x^30 + x^40 + 1)
```

Problem 18. Factor  $x^4 - 3x^2 + 1$  over the algebraic extension  $\mathbb{Q}(X)$  determined by the polynomial  $X^2 - X - 1$ .

Maxima:

```
(C18) factor( x^4 - 3*x^2 + 1, X^2 - X - 1 );
(D18) (x - X) (x - X + 1) (x + X - 1) (x + X)
```

MuPAD:

```
>> QX := Dom::AlgebraicExtension(Dom::Rational, X^2 - X - 1):
>> QX::Name := "QX": factor( poly(x^4 - 3*x^2 + 1, QX) )
poly(x + (X - 1), [x], QX) poly(x + X, [x], QX)
poly(x + (- X + 1), [x], QX) poly(x - X, [x], QX)
```

Problem 19. Factor  $x^4 - 3x^2 + 1$  modulo 5.

Maxima:

```
(C19) factor( x^4 - 3*x^2 + 1 ), modulus : 5;
(D19)          2          2
          (x - 2) (x + 2)
```

MuPAD:

```
>> factor( poly(x^4 - 3*x^2 + 1, IntMod(5)) )
          2          2
poly(x - 2, [x], IntMod(5)) poly(x + 2, [x], IntMod(5))
```

Problem 20. Compute the partial fraction decomposition of the rational expression  $(x^2 + 2x + 3)/(x^3 + 4x^2 + 5x + 2)$ .

Maxima:

```
(C20) partfrac( (x^2 + 2*x + 3)/(x^3 + 4*x^2 + 5*x + 2), x );
(D20)          3          2          2
          ----- - ----- + -----
          x + 2    x + 1          2
                                   (x + 1)
```

MuPAD:

```
>> partfrac( (x^2 + 2*x + 3)/(x^3 + 4*x^2 + 5*x + 2), x )
          2          2          3
          ----- - ----- + -----
          2    x + 1    x + 2
          (x + 1)
```

## 4 Inequalities

Problem 21. Is  $z = x$ , if  $x \geq y$  and  $y \geq z$  and  $z \geq x$ ?

Maxima:

```
(C21) assume(x >= y, y >= z, z >= x)$ is( x = z );
(C22)
(D22)          FALSE
```

MuPAD:

```
>> assume(x >= y): assume(y >= z, _and): assume(z >= x, _and):  
>> is( x = z )
```

TRUE

Problem 22. Is  $2x^2 > 2y^2$ , if  $x > y > 0$ ?

Maxima:

```
(C22) assume(x > y, y > 0)$ is( 2*x^2 > 2*y^2 );  
(C23)  
(D23) FALSE
```

MuPAD:

```
>> assume(x > y): assume(y > 0, _and): is( 2*x^2 > 2*y^2 )
```

TRUE

Problem 23. Solve the inequality  $|x - 1| > 2$ .

Maxima:

MuPAD:

```
>> solve( abs(x - 1) > 2 )  
  
x in ]-infinity, -1[ union ]3, infinity[
```

Problem 24. Expand and then solve the inequality  $(x - 1)(x - 2)(x - 3)(x - 4)(x - 5) < 0$ .

Maxima:

MuPAD:

```
>> solve(expand( (x - 1)*(x - 2)*(x - 3)*(x - 4)*(x - 5) < 0 ))  
  
x in ]2, 3[ union ]4, 5[ union ]-infinity, 1[
```

## 5 Trigonometry

Problem 25. Simplify the expression  $\cos(3x)/\cos(x)$ .

Maxima:

```
(C25) trigsimp( cos(3*x)/cos(x) ), trigexpand;
```

$$(D25) \quad \cos^2(x) - 3 \sin^2(x)$$

MuPAD:

```
>> expand( cos(3*x)/cos(x) )
```

$$\cos^2(x) - 3 \sin^2(x)$$

Problem 26. Linearize the expression  $\cos(3x)/\cos(x)$ .

Maxima:

```
(C26) trigrat(cos(3*x)/cos(x));
```

$$(D26) \quad 2 \cos(2x) - 1$$

MuPAD:

```
>> combine(expand( cos(3*x)/cos(x) ), sincos)
```

$$2 \cos(2x) - 1$$

Problem 27. Rewrite the expression  $\cos(3x)/\cos(x)$  using rewrite rules.

Maxima:

```
(C27) defrule( mcos, cos(3*x), trigexpand(cos(3*x)) )$
```

```
(C28) ratsimp(apply1( cos(3*x)/cos(x), mcos ));
```

$$(D28) \quad \cos^2(x) - 3 \sin^2(x)$$

MuPAD:

```
>> mcos := ( cos(3*x) = expand(cos(3*x)) ):
```

```
>> normal(subs( cos(3*x)/cos(x), mcos ))
```

$$\cos^2(x) - 3 \sin^2(x)$$

## 6 Finding zero

Problem 28. Simplify the expression  $\sqrt{997} - (997^3)^{1/6}$ .

Maxima:

```
(C28) sqrt(997) - (997^3)^(1/6);
(D28) 0
```

MuPAD:

```
>> simplify( sqrt(997) - (997^3)^(1/6) )
0
```

Problem 29. Simplify the expression  $\sqrt{999983} - (999983^3)^{1/6}$ .

Maxima:

```
(C29) float( sqrt(999983) - (999983^3)^(1/6) );
(D29) 0.0
```

MuPAD:

```
>> simplify( sqrt(999983) - (999983^3)^(1/6) )
0
```

Problem 30. Simplify  $(2^{1/3} + 4^{1/3})^3 - 6(2^{1/3} + 4^{1/3}) - 6$ .

Maxima:

```
(C30) radcan( (2^(1/3) + 4^(1/3))^3 - 6*(2^(1/3) + 4^(1/3)) - 6 );
(D30) 0
```

MuPAD:

```
>> radsimp( (2^(1/3) + 4^(1/3))^3 - 6*(2^(1/3) + 4^(1/3)) - 6 )
0
```

Problem 31. Simplify  $\log \tan(x/2 + \pi/4) - \sinh^{-1} \tan x$  to zero only if  $-\pi/2 < x < \pi/2$ .

Maxima:

```
(C31) declare(cos, posfun)$ trigexpand : true$
(C32)
(C33) trigsimp(trigreduce(logcontract(trigsimp(logarc(
log(tan(x/2+%pi/4)) - asinh(tan(x)) )))));
(D33) 0
```

MuPAD:

```
[See however the solution of the following problem.]
```

Problem 32. Show that the expression of the previous problem  $\log \tan(x/2 + \pi/4) - \sinh^{-1} \tan x$  is zero at  $x = 0$ , and its derivative is zero, if  $-\pi/2 < x < \pi/2$ .

Maxima:

```
(C32) f(x) := log(tan(x/2+%pi/4))-asinh(tan(x))$ subst(0,x,f(x));
(C33)
(D33)
0

(C34) assume(-%pi/2<x, x<%pi/2)$
(C35) diff(f(x), x), trigreduce;
(D35)
SEC(x) - COS(x) SEC(x)
2
(C36) trigsimp(D35);
(D36)
0
```

MuPAD:

```
>> f := ln(tan(x/2+PI/4)) - arcsinh(tan(x)): eval(subs(f, x = 0))
0

>> assume(x > -PI/2): assume(x < PI/2, _and):
>> simplify(expand(combine(normal(expand( diff(f, x) )),sincos)))
0
```

Problem 33. Simplify  $\log \frac{2\sqrt{r}+1}{\sqrt{4r+4\sqrt{r}+1}}$ .

Maxima:

```
(C33) logcontract(radcan(
log((2*sqrt(r)+1)/sqrt(4*r+4*sqrt(r)+1)) ));
(D33)
0
```

MuPAD:

```
>> assume(r>=0):
>> simplify( ln((2*sqrt(r)+1)/sqrt(4*r+4*sqrt(r)+1)) )
0
```

Problem 34. Simplify  $-2\sqrt{r} + (2\sqrt{r} + 1)^{\frac{1}{2\sqrt{r}+1}} (4r + 4\sqrt{r} + 1)^{\sqrt{r}\frac{1}{2\sqrt{r}+1}} - 1$ .

Maxima:

```
(C34) -2*sqrt(r)+(2*sqrt(r)+1)^(1/(2*sqrt(r)+1))*(4*r+4*sqrt(r)+1)
^(sqrt(r)/(2*sqrt(r)+1))-1, sqrt(r) = s, r = s^2, radcan;
(D34)
0
```

MuPAD:

```
>> assume(r>=0): f := -2*sqrt(r)+(2*sqrt(r)+1)^(1/(2*sqrt(r)+1))*
&> (4*r+4*sqrt(r)+1)^(sqrt(r)/(2*sqrt(r)+1))-1: simplify(f)
```

$$\frac{1}{(2r + 1)^{1/2}} + \frac{2r^{1/2}}{(2r + 1)^{1/2}} - 2r^{1/2} - 1$$

```
>> subsop(%, [2,2] = normal(op(%, [2,2])))
```

0

## 7 Complex domain

Problem 35. Represent  $\log(3 + 4i)$  in the rectangular form.

Maxima:

```
(C35) rectform( log(3 + 4*i) );
```

```
(D35) LOG(5) + %I ATAN(-)
      4
      3
```

MuPAD:

```
>> rectform( ln(3 + 4*I) )
```

$$\ln(5) + I \arctan(4/3)$$

Problem 36. Represent  $\tan(x + iy)$  in the rectangular form.

Maxima:

```
(C36) rectform( tan(x + %i*y) );
```

```
(D36) %I SINH(2 y) SIN(2 x)
      ----- + -----
      COSH(2 y) + COS(2 x) COSH(2 y) + COS(2 x)
```

MuPAD:

```
>> assume(x,Type::Real):assume(y,Type::Real):rectform(tan(x+I*y))
```

$$\frac{\sin(2x)}{\cos(2x) + \cosh(2y)} + \frac{\sinh(2y)}{\sqrt{\cos(2x) + \cosh(2y)}} I$$

Problem 37. Simplify  $\frac{\sqrt{xy|z|^2}}{\sqrt{x}|z|}$ .

Maxima:

```
(C37) declare([x, y, z], complex)$
(C38) ratsimp( sqrt(x*y*abs(z)^2)/(sqrt(x)*abs(z)) );
(D38)
      Sqrt(x y)
      -----
      Sqrt(x)
```

MuPAD:

```
>> ratsimp( sqrt(x*y*abs(z)^2)/(sqrt(x)*abs(z)) )
      1/2
      (x y)
      -----
      1/2
      x
```

Problem 38. Simplify  $\sqrt{1/z} - 1/\sqrt{z}$  to zero unless  $z < 0$ .

Maxima:

```
(C38) declare(z, complex)$ ratsimp( sqrt(1/z) - 1/sqrt(z) );
(C39)
(D39)
      0
```

MuPAD:

```
>> simplify( sqrt(1/z) - 1/sqrt(z) )
      / 1 \1/2    1
      | - |    - ----
      \ z /      1/2
                  z

>> assume(Re(z)>0): assume(Re(z)<0,_or): assume(Im(z)<>0,_and):
>> simplify( sqrt(1/z) - 1/sqrt(z) )
      0
```

Problem 39. Don't simplify the expression  $\sqrt{e^z} - e^{z/2}$  to zero without further assumptions.

Maxima:

```
(C39) declare(z, complex)$ sqrt(%e^z) - %e^(z/2);
(C40)
(D40) 0
```

MuPAD:

```
>> f := sqrt(exp(z)) - exp(z/2): simplify( f )
          1/2      / z \
exp(z)  - exp| - |
          \ 2 /

>> assume(Im(z) <= PI): assume(Im(z) > -PI, _and): simplify( f )
0
```

Problem 40. Simplify the expression  $\sqrt{e^{6i}}$ .

Maxima:

```
(C40) sqrt(%e^(6*i));
(D40) 3 %I
      %E
```

MuPAD:

```
>> sqrt(exp(6*I))
      -exp(3 I)
```

Problem 41. Simplify  $\log e^z$  to  $z$  only if  $-\pi < \text{Im } z \leq \pi$ .

Maxima:

```
(C41) declare(z, complex)$ log(%e^z);
(C42)
(D42) z
```

MuPAD:

```
>> simplify( ln(exp(z)) )
ln(exp(z))
>> assume(Im(z)>-PI):assume(Im(z)<=PI, _and):simplify(ln(exp(z)))
z
```

Problem 42. Simplify  $\log e^{10i}$  to  $(10 - 4\pi)i$ .

Maxima:

```
(C42) log(%e^(10*i));
(D42) 10 %I
```

MuPAD:

```
>> ln(exp(10*I))
10 I - 4 I PI
```

Problem 43. Simplify  $(xy)^{1/n} - x^{1/n}y^{1/n}$  to zero only with adequate assumptions on  $x$ ,  $y$  and  $n$ .

Maxima:

```
(C43) declare([x,y,n],complex)$radcan((x*y)^(1/n)-x^(1/n)*y^(1/n));
(C44)
(D44) 0
```

MuPAD:

```
>> simplify( (x*y)^(1/n) - x^(1/n)*y^(1/n) )
1 1 1
- - -
n n n
(x y) - x y
>> assume(n <> 0): assume(n, Type::Real, _and): assume(x > 0):
>> simplify( (x*y)^(1/n) - x^(1/n)*y^(1/n) )
0
```

Problem 44. Simplify  $\arctan \tan z$  to  $z$  only if  $-\pi < z \leq \pi/2$ .

Maxima:

```
(C44) declare(z, complex)$ atan(tan(z));
(C45)
(D45) z
```

MuPAD:

```
>> simplify( arctan(tan(z)) )
arctan(tan(z))
>> assume(z<=PI/2):assume(z>-PI/2,_and):simplify(arctan(tan(z)))
z
```

Problem 45. Simplify  $\arctan \tan 4$  to its correct value  $4 - \pi$ .

Maxima:

```
(C45) atan(tan(4));
(D45) 4
```

MuPAD:

```
>> arctan(tan(4))
4 - PI
```

## 8 Equations

Problem 46. Simplify  $(x = 0)/2 + 1$ .

Maxima:

```
(C46) (x = 0)/2 + 1;
(D46) x
- + 1 = 1
2
```

MuPAD:

```
>> (x = 0)/2 + 1
x
- + 1 = 1
2
```

Problem 47. Solve the equation  $3x^3 - 18x^2 + 33x - 19 = 0$ .

Maxima:

```
(C47) sol : solve( 3*x^3 - 18*x^2 + 33*x - 19 = 0 )$
(C48) trigreduce(trigsimp(rectform( sol )));
```

$$(D48) \left[ x = \frac{\sin\left(\frac{\operatorname{atan}\left(\frac{1}{\sqrt{3}}\right)}{3}\right) - \frac{\cos\left(\frac{\operatorname{atan}\left(\frac{1}{\sqrt{3}}\right)}{3}\right)}{\sqrt{3}} + 2, \right.$$

$$x = -\frac{\sin\left(\frac{\operatorname{atan}\left(\frac{1}{\sqrt{3}}\right)}{3}\right) - \frac{\cos\left(\frac{\operatorname{atan}\left(\frac{1}{\sqrt{3}}\right)}{3}\right)}{\sqrt{3}} + 2, \right.$$

$$x = \left. \frac{2 \cos\left(\frac{\operatorname{atan}\left(\frac{1}{\sqrt{3}}\right)}{3}\right)}{\sqrt{3}} + 2 \right]$$

MuPAD:

```
>> sol := solve(3*x^3 - 18*x^2 + 33*x - 19 = 0, x, MaxDegree = 3);
>> simplify(rectform( sol ))
```

$$\left\{ \frac{1}{2} \sqrt[3]{\frac{\cos\left(\frac{\sqrt{18}}{3}\right)}{\sqrt{18}}} + 2, \frac{1}{2} \sqrt[3]{\frac{\cos\left(\frac{\sqrt{18}}{3}\right)}{\sqrt{18}}} + 2, \right.$$

$$\left. -\frac{\sin\left(\frac{\sqrt{18}}{3}\right)}{\sqrt{18}} - \frac{\cos\left(\frac{\sqrt{18}}{3}\right)}{3} + 2 \right\}$$

Problem 48. Solve the equation  $x^4 + x^3 + x^2 + x + 1 = 0$ .

Maxima:

```
(C48) eq : x^4 + x^3 + x^2 + x + 1 = 0$ sol : solve( eq );
```

```
(C49)
```

$$(D49) \quad [x = -\frac{5^{1/4} \sqrt{\sqrt{5} - 1} \%I}{2 \sqrt{2}} - \frac{\sqrt{5}}{4} - \frac{1}{4},$$

$$x = \frac{5^{1/4} \sqrt{\sqrt{5} - 1} \%I}{2 \sqrt{2}} - \frac{\sqrt{5}}{4} - \frac{1}{4},$$

$$x = -\frac{5^{1/4} \sqrt{\sqrt{5} + 1} \%I}{2 \sqrt{2}} + \frac{\sqrt{5}}{4} - \frac{1}{4},$$

$$x = \frac{5^{1/4} \sqrt{\sqrt{5} + 1} \%I}{2 \sqrt{2}} + \frac{\sqrt{5}}{4} - \frac{1}{4}]$$

MuPAD:

```
>> eq := x^4 + x^3 + x^2 + x + 1 = 0:
>> sol := solve( eq, x, MaxDegree = 4 )
```

```
{
  {
    {
      {
        / 1/2      \1/2
        | 5
        | ----- - 5/2 |
        \ 2          /
      }
    }
  } ----- - 1/4,
  {
    4          2
  }

  / 1/2      \1/2
  | 5
  | ----- - 5/2 |
  \ 2          /
  ----- - 1/4,
  2          4

  / 1/2      \1/2
  | 5
  | ----- - 5/2 |
  \ 2          /
  ----- - 1/4,
  4          2

  / 1/2      \1/2 }
  | 5
  | ----- - 5/2 | }
  \ 2          / }
  ----- + ----- - 1/4 }
  4          2 }
}
```

Problem 49. Verify a solution of the previous problem.

Maxima:

```
(C49) eq, sol[1], radcan;
(D49) 0 = 0
```

MuPAD:

```
>> expand( subs(eq, x = sol[1]) )
0 = 0
```

Problem 50. Solve the equation  $e^{2x} + 2e^x + 1 = z$  for  $x$ .

Maxima:

```
(C50) declare(z,complex)$ solve( exp(2*x) + 2*exp(x) + 1 = z, x );
(C51)
(D51)      [x = LOG(- SQRT(z) - 1), x = LOG(SQRT(z) - 1)]
```

MuPAD:

```
>> solve( exp(2*x) + 2*exp(x) + 1 = z, x )
{ 2*I*X5*PI + ln(- z^(1/2) - 1) | X5 in Z_ } union piecewise(
  {} if z = 1, { 2*I*X8*PI + ln(z^(1/2) - 1) | X8 in Z_ } if
  z <> 1)
```

Problem 51. Solve the equation  $(x + 1)(\sin^2 x + 1)^2 \cos^3 3x = 0$ .

Maxima:

```
(C51) solve( (x + 1)*(sin(x)^2 + 1)^2*cos(3*x)^3 = 0 );
```

SOLVE is using arc-trig functions to get a solution.  
Some solutions will be lost.

```
(D51)      %PI
          [x = ----, x = - 1]
           6
```

MuPAD:

```
>> solve( (x + 1)*(sin(x)^2 + 1)^2*cos(3*x)^3 = 0 )
x in {-1} union { PI + X3*PI - I*arcsinh(1) | X3 in Z_ } union
  { I*arcsinh(1) - X4*PI | X4 in Z_ } union
  { 1/6*PI + 1/3*X2*PI | X2 in Z_ }
```

Problem 52. Solve the equation  $e^z = 1$ .

Maxima:

```
(C52) declare(z, complex)$ solve( %e^z = 1 );
(C53)
(D53)      [z = 0]
```

MuPAD:

```
>> solve( E^z = 1 )
```

```
z in { 2*I*X1*PI | X1 in Z_ }
```

Problem 53. Solve the equation  $\sin x = \cos x$ .

Maxima:

```
(C53) solve( sin(x) = cos(x) );
```

```
(D53) [SIN(x) = COS(x)]
```

MuPAD:

```
>> solve( sin(x) = cos(x) )
```

```
x in { 1/4*PI + X2*PI | X2 in Z_ }
```

Problem 54. Solve the equation  $\tan x = 1$ .

Maxima:

```
(C54) solve( tan(x) = 1 );
```

SOLVE is using arc-trig functions to get a solution.  
Some solutions will be lost.

```
(D54) [x = ---]
      %PI
      4
```

MuPAD:

```
>> solve( tan(x) = 1 )
```

```
x in { 1/4*PI + X2*PI | X2 in Z_ }
```

Problem 55. Solve the equation  $\sin x = \tan x$ .

Maxima:

```
(C55) solve( sin(x) = tan(x) );
```

SOLVE is using arc-trig functions to get a solution.  
Some solutions will be lost.

```
(D55) [x = 0]
```

MuPAD:

```
>> solve( sin(x) = tan(x) )
```

```
x in { X1*PI | X1 in Z_ }
```

Problem 56. Solve the equation  $\sqrt{x^2 + 1} = x - 2$ .

Maxima:

```
(C56) solve( sqrt(x^2 + 1) = x - 2 );  
(D56) [x = SQRT(x^2 + 1) + 2]
```

MuPAD:

```
>> solve( sqrt(x^2 + 1) = x - 2 )  
  
{ }
```

Problem 57. Find the real roots of the equation  $e^{2-x^2} = e^{-x}$ .

Maxima:

```
(C57) solve( %e^(2 - x^2) = %e^(-x) );  
(D57) [x = 2, x = - 1]
```

MuPAD:

```
>> assume(x, Type::Real): solve( exp(2 - x^2) = exp(-x) )  
  
{ [x = -1], [x = 2] }
```

Problem 58. Solve the equation  $\sqrt{\log x} = \log \sqrt{x}$ .

Maxima:

```
(C58) solve( sqrt(log(x)) = log(sqrt(x)) );  
(D58) [LOG(x) = 2 SQRT(LOG(x))]
```

MuPAD:

```
>> solve( sqrt(ln(x)) = ln(sqrt(x)) )  
  
{ [x = 1], [x = exp(4)] }
```

Problem 59. Solve the equation  $|x - 1| = 2$ .

Maxima:

```
(C59) solve( abs(x - 1) = 2 );  
(D59) [ABS(x - 1) = 2]
```

MuPAD:

```
>> assume(x, Type::Real): solve( abs(x - 1) = 2 )
      { [x = -1], [x = 3] }
```

Problem 60. Solve the system of linear equations

$$x + y + z = 6, 2x + y + 2z = 10, x + 3y + z = 10.$$

Maxima:

```
(C60) e1:  x +  y +  z = 6$
(C61) e2: 2*x +  y + 2*z = 10$
(C62) e3:  x + 3*y +  z = 10$
(C63) solve( [e1, e2, e3], [x, y, z] );
```

Dependent equations eliminated: (3)

```
(D63)      [[x = 4 - %R1, y = 2, z = %R1]]
```

MuPAD:

```
>> e1 :=  x +  y +  z = 6:
>> e2 := 2*x +  y + 2*z = 10:
>> e3 :=  x + 3*y +  z = 10:
>> solve( [e1, e2, e3], [x, y, z] )
      { [x = 4 - z, y = 2] }
```

Problem 61. Solve the system of nonlinear equations  $x^2 y + 3y z - 4 = 0$ ,  $-3x^2 z + 2y^2 + 1 = 0$ ,  $2y z^2 - z^2 - 1 = 0$ .

Maxima:

```

(C61) e1: x^2*y + 3*y*z - 4 = 0$
(C62) e2: -3*x^2*z + 2*y^2 + 1 = 0$
(C63) e3: 2*y*z^2 - z^2 - 1 = 0$
(C64) solve( [e1, e2, e3], [x, y, z] );
(D64) [[x = 1, y = 1, z = 1], [x = - 1, y = 1, z = 1],

```

$$[x = \text{SQRT}(3) \text{SQRT}\left(-\frac{4 \text{SQRT}(2) \%I}{11 \text{SQRT}(2) \%I - 1} - \frac{7}{11 \text{SQRT}(2) \%I - 1}\right),$$

$$y = \frac{\text{SQRT}(2) \%I + 4}{2 \text{SQRT}(2) \%I - 1}, z = \frac{\text{SQRT}(2) \%I + 1}{3}],$$

$$[x = - \text{SQRT}(3) \text{SQRT}\left(-\frac{4 \text{SQRT}(2) \%I}{11 \text{SQRT}(2) \%I - 1} - \frac{7}{11 \text{SQRT}(2) \%I - 1}\right),$$

$$y = \frac{\text{SQRT}(2) \%I + 4}{2 \text{SQRT}(2) \%I - 1}, z = \frac{\text{SQRT}(2) \%I + 1}{3}],$$

$$[x = \text{SQRT}(3) \text{SQRT}\left(\frac{7}{11 \text{SQRT}(2) \%I + 1} - \frac{4 \text{SQRT}(2) \%I}{11 \text{SQRT}(2) \%I + 1}\right),$$

$$y = \frac{\text{SQRT}(2) \%I - 4}{2 \text{SQRT}(2) \%I + 1}, z = -\frac{\text{SQRT}(2) \%I - 1}{3}],$$

$$[x = - \text{SQRT}(3) \text{SQRT}\left(\frac{7}{11 \text{SQRT}(2) \%I + 1} - \frac{4 \text{SQRT}(2) \%I}{11 \text{SQRT}(2) \%I + 1}\right),$$

$$y = \frac{\text{SQRT}(2) \%I - 4}{2 \text{SQRT}(2) \%I + 1}, z = -\frac{\text{SQRT}(2) \%I - 1}{3}],$$

$$[x = .5332208394853406, y = .6171928620452986,$$

$$z = 2.065544197233915], [x = - .5332208394853406,$$

$$y = .6171928620452986, z = 2.065544197233915],$$

```

[x = .7577068583503752 %I + 1.800995670082093,
y = 1.069034852482791 - 1.481223286209397 %I,
z = - .3178876919352108 %I - .4626596394477437],
[x = - .7577068583503704 %I - 1.800995670082076,
y = 1.069034852482791 - 1.481223286209397 %I,
z = - .3178876919352108 %I - .4626596394477437],
[x = 1.800995670082078 - 0.757706858350376 %I,
y = 1.481223286209397 %I + 1.069034852482791,
z = .3178876919352108 %I - .4626596394477437],
[x = .7577068583503762 %I - 1.800995670082078,
y = 1.481223286209397 %I + 1.069034852482791,
z = .3178876919352108 %I - .4626596394477437],
[x = 1.688956979562167 %I + .7354914914397239,
y = - .5358586759039574 %I - 1.377631273618585,
z = - .5011518608929353 %I - .0701123791217747],
[x = - 1.688956979562165 %I - .7354914914397243,
y = - .5358586759039574 %I - 1.377631273618585,
z = - .5011518608929353 %I - .0701123791217747],
[x = 0.735491491439724 - 1.688956979562167 %I,
y = .5358586759039574 %I - 1.377631273618585,
z = .5011518608929353 %I - .0701123791217747],
[x = 1.688956979562167 %I - .7354914914397239,
y = .5358586759039574 %I - 1.377631273618585,
z = .5011518608929353 %I - .0701123791217747]]

```

MuPAD:

```

>> e1 := x^2*y + 3*y*z - 4 = 0:
>> e2 := -3*x^2*z + 2*y^2 + 1 = 0:
>> e3 := 2*y*z^2 - z^2 - 1 = 0:
>> solve( [e1, e2, e3], [x, y, z] )

{
{
{ [x = -1, y = 1, z = 1], [x = 1, y = 1, z = 1],
{
}
}
}

--      1/2      2      3      4 1/2
|      3      (30 z  - 7 z + 12 z  - 12 z )
| x = -----,
--      3

      2
      21 z      3      4
y = 6 z + ---- + 12 z  - 9 z  + 3/2,
      2

z = RootOf(6 X15 - 7 X12 - 9 X13 - 6 X14 - 3 X1 - 1, X1) |,
--

--      1/2      2      3      4 1/2
|      3      (30 z  - 7 z + 12 z  - 12 z )
| x = -----,
--      3

      2
      21 z      3      4
y = 6 z + ---- + 12 z  - 9 z  + 3/2,
      2

z = RootOf(6 X15 - 7 X12 - 9 X13 - 6 X14 - 3 X1 - 1, X1) |,
--

[x = (- I 21/2 - 1)1/2, y = I 21/2, z = 1/3 - 1/3 I 21/2 ],
[x = (I 21/2 - 1)1/2, y = - I 21/2, z = 1/3 I 21/2 + 1/3],

```

$$\begin{aligned}
 & \left[ x = - \left( - \frac{1}{2} \frac{1}{2} - 1 \right) \frac{1}{2}, y = \frac{1}{2} \frac{1}{2}, z = \frac{1}{3} - \frac{1}{3} \frac{1}{2} \right], \\
 & \left. \begin{aligned}
 & \left[ x = - \left( \frac{1}{2} \frac{1}{2} - 1 \right) \frac{1}{2}, y = - \frac{1}{2} \frac{1}{2}, z = \frac{1}{3} \frac{1}{2} + \frac{1}{3} \right] \end{aligned} \right\}
 \end{aligned}$$

### 9 Matrix

Problem 62. Invert the 2 by 2 matrix  $\begin{bmatrix} a & b \\ 1 & a \cdot b \end{bmatrix}$ .

Maxima:

```

(C62) matrix( [a, b], [1, a*b] )^-1;
      [      a          1      ]
      [  -----  -  ----- ]
      [      2          2      ]
      [  a - 1      a - 1      ]
(D62) [      ]
      [      ]
      [      1          a      ]
      [ - -----  - ----- ]
      [      2          2      ]
      [  (a - 1) b  (a - 1) b  ]
  
```

MuPAD:

```

>> 1/matrix( [[a, b], [1, a*b]] )
      +-          +-
      |      a          1      |
      |  -----,  -  ----- |
      |      2          2      |
      |  a - 1      a - 1      |
      |      |
      |      1          a      |
      | - -----,  - ----- |
      |      2          2      |
      |  - b + a b  - b + a b  |
      |      |
      +-          +-
  
```

Problem 63. Compute the determinant of Vandermonde-matrix for variables  $w$ ,  $x$ ,  $y$  and  $z$ .

Maxima:

```
(C63) Vandermonde(list) := block(local(line,Vm),
line[i] := list^(i-1), Vm[i,j] := line[i][j],
genmatrix(Vm,length(list),length(list)))$
(C64) factor(determinant(Vandermonde( [w, x, y, z] )));
(D64)      (x - w) (y - w) (y - x) (z - w) (z - x) (z - y)
```

MuPAD:

```
>> Vandermonde := list -> matrix([_seqgen(map(list,_power,i-1), i,
&> 1..nops(list))]);
>> factor(linalg::det(Vandermonde( [w, x, y, z] )))

      (- w + x) (- w + y) (- x + y) (w - z) (x - z) (- y + z)
```

Problem 64. Find the eigenvalues of the 3 by 3 matrix  $[[5, -3, -7], [-2, 1, 2], [2, -3, -4]]$ .

Maxima:

```
(C64) eigenvalues(matrix( [5,-3,-7], [-2,1,2], [2,-3,-4] ));
Warning - you are redefining the MACSYMA function EIGENVALUES
Warning - you are redefining the MACSYMA function EIGENVECTORS
(D64)      [[3, - 2, 1], [1, 1, 1]]
```

MuPAD:

```
>> linalg::eigenvalues(matrix( [[5,-3,-7], [-2,1,2], [2,-3,-4]] ))

      {-2, 1, 3}
```

Problem 65. Calculation of a tensor covariant derivative.

Maxima:

MuPAD:

Problem 66. Prove the Bianchi property for tensors.

Maxima:

MuPAD:

## 10 Sums and products

Problem 67. Calculate the sum  $\sum_{k=1}^n k^3$ .

Maxima:

```
(C67) sum(k^3, k, 1, n), factor, simpsum;
```

$$(D67) \quad \frac{n^2 (n+1)^2}{4}$$

MuPAD:

```
>> factor( sum(k^3, k = 1 .. n) )
```

$$1/4 n^2 (n+1)^2$$

Problem 68. Calculate the sum  $\sum_{k=1}^{\infty} \frac{1}{k^2} + \frac{1}{k^3}$ .

Maxima:

```
(C68) sum(1/k^2 + 1/k^3, k, 1, inf), simpsum;
```

$$(D68) \quad \text{ZETA}(3) + \frac{\%PI^2}{6}$$

MuPAD:

```
>> sum(1/k^2 + 1/k^3, k = 1 .. infinity)
```

$$\text{zeta}(3) + \frac{\text{PI}^2}{6}$$

Problem 69. Approximate the sum of the previous problem.

Maxima:

```
(C69) sol : ev( sum(1/k^2 + 1/k^3, k, 1, inf), simpsum, numer )$
(C70) load(bffac)$ ev( sol, zeta(3) = bfzeta(3,16) );
(C71)
Warning: Float to bigfloat conversion of 1.6449340668482262
(D71) 2.84699097000782B0
```

MuPAD:

```
>> float( sum(1/k^2 + 1/k^3, k = 1 .. infinity) )
2.84699097
```

Problem 70. Calculate the product  $\prod_{k=1}^n k$ .

Maxima:

```
(C70) product(k, k, 1, n);
(D70) n!
```

MuPAD:

```
>> rewrite( product(k, k = 1..n), fact );
fact(n)
```

## 11 Limits

Problem 71. Calculate the limits  $\lim_{n \rightarrow \infty} (1 + 1/n)^n$  and  $\lim_{x \rightarrow 0} (1 - \cos x)/x^2$ .

Maxima:

```
(C71) limit( (1 + 1/n)^n, n, inf );
(D71) %E

(C72) limit( (1 - cos(x))/x^2, x, 0 );
1
(D72) -
2
```

MuPAD:

```
>> limit( (1 + 1/n)^n, n = infinity )
                exp(1)
>> limit( (1 - cos(x))/x^2, x = 0 )
                1/2
```

Problem 72. Calculate the second derivative of the expression  $y(x(t))$  with respect to  $t$ .

Maxima:

```
(C72) depends(y, x, x, t)$ diff(y, t, 2);
(C73)
```

```
(D73)
                2      2
            dx 2 d y   d x dy
            (---) --- + --- --
            dt      2      2 dx
                dx      dt
```

MuPAD:

```
>> diff( y(x(t)), t, t )
                2
            D(y)(x(t)) diff(x(t), t, t) + D(D(y))(x(t)) diff(x(t), t)
```

## 12 Integration

Problem 73. Integrate  $1/(x^3 + 2)$ . Differentiate the result to get back the integrand.

Maxima:

(C73) i73 : integrate( 1/(x^3 + 2), x );

$$\begin{aligned}
 & \text{ATAN}\left(\frac{\sqrt[3]{2x-2}}{\sqrt[3]{2}\sqrt{3}}\right) \\
 (D73) \quad & - \frac{\text{LOG}(x^{\frac{2}{3}} - 2^{\frac{1}{3}} x^{\frac{2}{3}})}{6^{\frac{2}{3}}} + \frac{\text{LOG}(x + 2^{\frac{1}{3}})}{3^{\frac{2}{3}}}
 \end{aligned}$$

(C74) radcan( diff(i73, x) );

$$(D74) \quad \frac{1}{x^3 + 2}$$

MuPAD:



MuPAD:

```
>> assume(Type::Real): assume(b>abs(a)): assume(b<-abs(a), _or):
>> i74 := int( 1/(a + b*cos(x)), x )
```

$$\frac{1}{\sqrt{b^2 - a^2}} \ln \left| \frac{a + b \cos(x) - \sqrt{b^2 - a^2} \sin(x)}{a + b \cos(x) + \sqrt{b^2 - a^2} \sin(x)} \right| - \frac{1}{\sqrt{b^2 - a^2}} \frac{a + b \cos(x) - \sqrt{b^2 - a^2} \sin(x)}{a + b \cos(x) + \sqrt{b^2 - a^2} \sin(x)}$$

$$\frac{1}{\sqrt{b^2 - a^2}} \ln \left| \frac{a + b \cos(x) - \sqrt{b^2 - a^2} \sin(x)}{a + b \cos(x) + \sqrt{b^2 - a^2} \sin(x)} \right| - \frac{1}{\sqrt{b^2 - a^2}} \frac{a + b \cos(x) - \sqrt{b^2 - a^2} \sin(x)}{a + b \cos(x) + \sqrt{b^2 - a^2} \sin(x)}$$

Problem 75. Differentiate the result i74 of the previous problem to find back the integrand  $1/(a + b \cos x)$ .

Maxima:

```
(C75) trigsimp(ratsimp( diff( i74, x ) ));
```

(D75)

$$\frac{1}{b \cos(x) + a}$$

MuPAD:

```
>> normal(subs(normal(expand(combine(normal(
&> diff(i74, x)), sincos))), sin(x)^2 = 1 - cos(x)^2) )
```

$$\frac{1}{a + b \cos(x)}$$

Problem 76. Differentiate the absolute function.

Maxima:

```
(C76) diff( abs(x), x );
```

```
(D76)          x
          -----
          ABS(x)
```

MuPAD:

```
>> diff( abs(x), x )
```

```
sign(x)
```

Problem 77. Integrate the absolute function.

Maxima:

```
(C77) integrate( abs(x), x );
```

```
(D77)          /
          [
          I ABS(x) dx
          ]
          /
```

MuPAD:

```
>> int( abs(x), x )
```

```
      2
     x sign(x)
     -----
          2
```

Problem 78. Define the absolute function piecewise and differentiate it.

Maxima:

```
(C78) a(t) := if t < 0 then -t else t$ diff( a(x), x );
```

```
(C79)
```

```
MACSYMA was unable to evaluate the predicate:
```

```
x < 0
```

```
#0: a(t=x)
```

```
-- an error. Quitting. To debug this try DEBUGMODE(TRUE);)
```

MuPAD:

```
>> a := piecewise([x < 0, -x], [x >= 0, x]): diff( a(x), x )
```

```
piecewise(-1 if x < 0, 1 if 0 < x)
```

Problem 79. Define the absolute function piecewise and integrate it.

Maxima:

```
(C79) a(t) := if t < 0 then -t else t$ integrate( a(x), x );
(C80)
MACSYMA was unable to evaluate the predicate:
x < 0
#0: a(t=x)
-- an error. Quitting. To debug this try DEBUGMODE(TRUE);)
```

MuPAD:

```
>> a := piecewise([x < 0, -x], [x >= 0, x]): int( a(x), x )

      /      2      2      \
      |      x      x      |
piecewise| - -- if x < 0, -- if 0 <= x |
      \      2      2      /
```

Problem 80. Integrate  $\int \frac{x}{\sqrt{x+1}+\sqrt{1-x}} dx$ .

Maxima:

```
(C80) i80 : integrate( x/(sqrt(1 + x) + sqrt(1 - x)), x );
      /
      [
      x
(D80)  I ----- dx
      ] SQRT(x + 1) + SQRT(1 - x)
      /
```

MuPAD:

```
>> i80 := int( x/(sqrt(1 + x) + sqrt(1 - x)), x )

      1/2 / x      \      1/2 /      x \
(x + 1) | - + 1/3 | + (1 - x) | 1/3 - - |
      \ 3      /      \      3 /
```

Problem 81. Integrate  $\int \frac{\sqrt{x+1}-\sqrt{1-x}}{2} dx$ , and show that it is same as the solution i80 of the previous Problem.

Maxima:

```
(C81) ratsimp( integrate( (sqrt(1+x) - sqrt(1-x))/2, x ) );
      (- x - 1) SQRT(x + 1) + SQRT(1 - x) (x - 1)
(D81)  -----
      3
```

MuPAD:

```
>> i81 := int( (sqrt(1 + x) - sqrt(1 - x))/2, x )
```

$$\frac{(x + 1)^{3/2}}{3} + \frac{(1 - x)^{3/2}}{3}$$

```
>> is( expand(i80) = simplify(i81) )
```

TRUE

### 13 Definite integrals

Problem 82. Determine the Cauchy principal value of the integral  $\int_{a-1}^{a+1} \frac{1}{x-a} dx$ .

Maxima:

```
(C82) integrate( 1/(x - a), x, a - 1, a + 1 );
```

Principal Value

```
(D82) 0
```

MuPAD:

```
>> int( 1/(x - a), x = a - 1 .. a + 1 )
```

undefined

```
>> int( 1/(x - a), x = a - 1 .. a + 1, PrincipalValue )
```

0

Problem 83. Show that the integral  $\int_{a-1}^{a+1} \frac{1}{(x-a)^2} dx$  is divergent.

Maxima:

```
(C83) integrate( 1/(x - a)^2, x, a - 1, a + 1 );
```

Integral is divergent

```
-- an error. Quitting. To debug this try DEBUGMODE(TRUE);)
```

MuPAD:

```
>> assume(a, Type::Real): int( 1/(x - a)^2, x = a - 1 .. a + 1 )
```

undefined

Problem 84. Calculate the integral  $\int_0^1 \sqrt{x + \frac{1}{x} - 2} dx$ .

Maxima:

```
(C84) integrate( sqrt(x + 1/x - 2), x, 0, 1 );
      4
(D84)  - -
      3
```

MuPAD:

```
>> numeric::int( sqrt(x + 1/x - 2), x = 0 .. 1 ):
>> numeric::rationalize( %, Restore )

      4/3
```

Problem 85. Calculate the integral  $\int_1^2 \sqrt{x + \frac{1}{x} - 2} dx$ .

Maxima:

```
(C85) integrate( sqrt(x + 1/x - 2), x, 1, 2 );
      4 2 Sqrt(2)
(D85)  - - -----
      3      3
```

MuPAD:

```
>> numeric::int( sqrt(x + 1/x - 2), x = 1 .. 2 )

      0.3905242918
```

Problem 86. Calculate the integral  $\int_0^2 \sqrt{x + \frac{1}{x} - 2} dx$ .

Maxima:

```
(C86) integrate( sqrt(x + 1/x - 2), x, 0, 2 );
      2 Sqrt(2)
(D86)  - -----
      3
```

MuPAD:

```
>> numeric::int( sqrt(x + 1/x - 2), x = 0 .. 2 )

      1.723857625
```

Problem 87. Calculate the integral  $\int_{-\infty}^{\infty} \frac{\cos x}{x^2+a^2} dx$  for  $a > 0$ .

Maxima:

```
(C87) assume(a>0)$ integrate( cos(x)/(x^2 + a^2), x, -inf, inf );
```

```
(C88)
```

```
(D88) 
$$\frac{\pi e^{-a}}{a}$$

```

MuPAD:

```
>> assume(a > 0):
```

```
>> simplify( int( cos(x)/(x^2 + a^2), x = -infinity..infinity ) )
```

```

$$\frac{\pi \exp(-a)}{a}$$

```

Problem 88. Calculate the integral  $\int_0^\infty \frac{t^{a-1}}{t+1} dt$  for  $0 < a < 1$ .

Maxima:

```
(C88) assume(a > 0)$ assume(a - 1 < 0)$ declare(a, noninteger)$
```

```
(C89)
```

```
(C90)
```

```
(C91) integrate( t^(a-1)/(1 + t), t, 0, inf );
```

```
(D91) 
$$\text{BETA}(a, 1 - a)$$

```

MuPAD:

```
>> assume(a > 0): assume(a < 1, _and):
```

```
>> int( t^(a-1)/(1 + t), t = 0..infinity )
```

```

$$\frac{\pi}{\sin(a \pi)}$$

```

Problem 89. Calculate the integral  $\int_0^\infty \frac{J_1(x)^2}{x^2} dx$  for the Bessel function  $J_1$ .

Maxima:

```
(C89) integrate( (bessel_j[1](x)/x)^2, x, 0, inf );
```

```
(D89) 
$$\int_0^{\infty} \frac{\text{BESSEL\_J}(x)^2}{x^2} dx$$

```

MuPAD:

```
>> int( (besselJ(1, x)/x)^2, x = 0..infinity )
```

$$\frac{4}{3 \text{ PI}}$$

Problem 90. Calculate the integral  $\int_0^a \int_0^{b(1-\frac{x}{a})} \int_0^{c((1-\frac{x}{a})-\frac{y}{b})} 1 dz dy dx$ .

Maxima:

```
(C90) integrate(integrate(integrate(
1, z, 0, c*(1-x/a-y/b)), y, 0, b*(1-x/a)), x, 0, a );
```

```
(D90)
      a b c
-----
      6
```

MuPAD:

```
>> int(int(int( 1, z=0..c*(1-x/a-y/b)), y=0..b*(1-x/a)), x=0..a )
```

$$\frac{a b c}{6}$$

## 14 Series

Problem 91. Find the Taylor expansion for  $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$  about  $v = 0$ .

Maxima:

```
(C91) t91 : taylor( 1/sqrt(1 - (v/c)^2), v, 0, 7 );
```

```
(D91)/T/
      2      4      6
      v      3 v      5 v
1 + ---- + ---- + ---- + . . .
      2      4      6
      2 c      8 c      16 c
```

MuPAD:

```
>> t91 := series( 1/sqrt(1 - (v/c)^2), v, 7 )
```

$$1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \frac{5v^6}{16c^6} + O(v^7)$$

Problem 92. Obtain the inverse of the square of the expansion t91 of the previous problem.

Maxima:

```
(C92) 1/t91^2;
(D92)/T/
```

$$1 - \frac{v^2}{c^2} + \dots$$

MuPAD:

```
>> 1/t91^2
```

$$1 - \frac{v^2}{c^2} + O(v^7)$$

Problem 93. Show that the Taylor expansion for tan(x) about x = 0 is obtained by dividing the expansion for sin(x) by the expansion for cos(x) about x = 0.

Maxima:

```
(C93) tsc : taylor(sin(x), x, 0, 7)/taylor(cos(x), x, 0, 7);
(D93)/T/
```

$$x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$$

```
(C94) ttan : taylor(tan(x), x, 0, 7);
(D94)/T/
```

$$x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$$

```
(C95) is( tsc = ttan );
(D95) TRUE
```

MuPAD:

```
>> tsc := series(sin(x), x, 7)/series(cos(x), x, 7)
```

$$x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + 0(x^9)$$

```
>> ttan := series(tan(x), x, 7)
```

$$x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + 0(x^9)$$

```
>> is( tsc = ttan )
```

TRUE

Problem 94. Find the Taylor expansion for  $(\log x)^a e^{-b x}$  about  $x = 1$ .

Maxima:

```
(C94) taylor( log(x)^a*exp(-b*x), x, 1, 1 );
```

$$(D94)/T/ \frac{(x-1)^a}{\%E^b} - \frac{((x-1)^a + 2b(x-1)^a)(x-1)^a}{2\%E^b} + \dots$$

MuPAD:

```
>> series( ln(x)^a*exp(-b*x), x = 1, 2 )
```

$$(x-1)^a \exp(-b) + (x-1)^2 \frac{a \exp(-b)}{2} - b \exp(-b) (x-1) + 0((x-1)^2 (x-1)^a)$$

Problem 95. Find the Taylor expansion for  $\log \sinh z + \log \cosh(z+w)$  about  $z = 0$ .

Maxima:

```
(C95) declare(z,complex, w,complex)$
(C96) t1s: taylor(log(sinh(z)) + log(cosh(z + w)), z, 0, 1);
(D96)/T/ LOG(z) + LOG(COSH(w)) + . . . +  $\frac{\text{SINH}(w) z}{\text{COSH}(w)}$  + . . .
```

MuPAD:

```
>> t1s := series(ln(sinh(z)) + ln(cosh(z + w)), z, 3)

(ln(z) + ln(cosh(w))) +  $\frac{z \sinh(w)}{\cosh(w)}$  + z  $\frac{\sinh^2(w)}{2 \cosh^2(w)}$  + 2/3 z2 +
O(z3)
```

Problem 96. Subtract from the expansion `t1s` of the previous problem the Taylor expansion for  $\log(\sinh z \cosh(z+w))$  about  $z = 0$ .

Maxima:

```
(C96) declare(z, complex, w, complex)$
(C97) t1s - taylor(log(sinh(z)*cosh(z + w)), z, 0, 1);
(D97)/T/ 0 + . . .
```

MuPAD:

```
>> t1s - series(ln(sinh(z)*cosh(z + w)), z, 2)

(ln(z) + ln(cosh(w)) - ln(z cosh(w))) + O(z2)

>> assume(z > 0): t1s - series(ln(sinh(z)*cosh(z + w)), z, 2)

O(z2)
```

Problem 97. Find the Taylor expansion for  $\log(\sin x/x)$  about  $x = 0$ .

Maxima:

```
(C97) taylor(log(sin(x)/x), x, 0, 7);
(D97)/T/ -  $\frac{x^2}{6}$  -  $\frac{x^4}{180}$  -  $\frac{x^6}{2835}$  + . . .
```

MuPAD:

```
>> series( ln(sin(x)/x), x, 6 )
```

$$-\frac{x^2}{6} - \frac{x^4}{180} - \frac{x^6}{2835} + O(x^8)$$

Problem 98. Find the general power series for  $\log(\sin x/x)$  about  $x = 0$ .

Maxima:

```
(C98) powerseries( log(sin(x)/x), x, 0 );
```

$$\frac{\sum_{I1=1}^{\infty} \frac{(-1)^{I1} \text{BERN}(2 I1) x^{2 I1}}{I1 (2 I1)!}}{I1 = 1}$$

(D98)

2

MuPAD:

```
>> series( ln(sin(x)/x), x, infinity )
```

$$\text{series} \left( \ln \left| \frac{\sin(x)}{x} \right|, x, \text{infinity} \right)$$

Problem 99. Find the general power series for  $e^{-x} \sin x$  about  $x = 0$ .

Maxima:

```
(C99) powerseries( exp(-x)*sin(x), x, 0 );
```

$$\sum_{I2=0}^{\infty} \frac{(-1)^{I2} x^{2 I2 + 1}}{(2 I2 + 1)!} - \sum_{I2=0}^{\infty} \frac{(-1)^{I2} x^{2 I2}}{(2 I2)!}$$

(D99)

I2 = 0

I2 = 0

MuPAD:

```
>> series( exp(-x)*sin(x), x, infinity )
```

$$\sum_{n1=0}^{\infty} \frac{x^{n1} \sin\left(\frac{\sqrt{3} \text{PI } n1}{4}\right)^2}{\text{fact}(n1)}$$

Problem 100. Revert the Taylor expansion for  $\sin y + \cos y$  about  $y = 0$ .

Maxima:

MuPAD:

```
>> revert( series( sin(y) + cos(y), y ) )
```

$$(y - 1) + \frac{(y - 1)^2}{2} + \frac{2(y - 1)^3}{3} + (y - 1)^4 + \frac{17(y - 1)^5}{10} + O((y - 1)^6)$$

Problem 101. Construct the Pade approximation of order two for  $e^{-x}$  at  $x = 0$ .

Maxima:

```
(C101) pade(taylor( exp(-x), x, 0, 2 ), 1, 1);
```

```
(D101) [- ----]
          x - 2
          x + 2
```

MuPAD:

```
>> pade( exp(-x), x, [1,1] )
```

$$\frac{2 - x}{x + 2}$$

## 15 Transforms

Problem 102. Find the Laplace transform for  $\cos((w - 1)t)$  with respect to  $t$ .

Maxima:

```
(C102) la102 : laplace( cos((w - 1)*t), t, s );
```

```
(D102)          s
          -----
                2    2
              (w - 1) + s
```

MuPAD:

```
>> la102 := transform::laplace( cos((w - 1)*t), t, s )
```

```
          s
          -----
                2    2
              s + (w - 1)
```

Problem 103. Obtain the inverse Laplace transform for the result `la102` of the previous problem.

Maxima:

```
(C103) ilt( la102, s, t );
```

```
Is w - 1 zero or nonzero?
```

```
nonzero;
```

```
(D103)          COS(t (w - 1))
```

MuPAD:

```
>> assume(w, Type::Real):
```

```
>> simplify( transform::invlaplace( la102, s, t ) )
```

```
cos(t (w - 1) sign(w - 1))
```

## 16 Differential equations

Problem 104. Solve the difference equation  $r_{n+2} - 2r_{n+1} + r_n = 2$  with the initial conditions  $r_0 = -1$  and  $r_1 = m$ .

Maxima:

```
(C104) load(recur)$ char( r(n+2) - 2*r(n+1) + r(n), 2, r, n, 2,
(C105) [r(0) = -1, r(1) = m] );
```

Dependent equations eliminated: (1 2)

$$(D105) \quad r(n) = n^2 + m n - 1$$

MuPAD:

```
>> re := r(n+2) - 2*r(n+1) + r(n) = 2: ic := {r(0)=-1, r(1)=m}:
>> solve( rec(re, r(n), ic) )
```

$$\{m n + n^2 - 1\}$$

Problem 105. Solve the initial-value problem  $f''(t) + 4f(t) = \sin(2t)$ ,  $f(0) = f'(0) = 0$ .

Maxima:

```
(C105) ivp : diff(f(t), t, 2) + 4*f(t) = sin(2*t)$
(C106) atvalue(f(t), t = 0, 0)$
(C107) atvalue(diff(f(t), t), t = 0, 0)$ desolve( ivp, f(t) );
(C108)
```

$$(D108) \quad f(t) = \frac{\sin(2 t)}{8} - \frac{t \cos(2 t)}{4}$$

MuPAD:

```
>> ivp := ode({f''(t) + 4*f(t) = sin(2*t), f(0)=0, f'(0)=0}, f(t)):
>> solve( ivp )
```

$$\left\{ \frac{\sin(2 t)}{8} - \frac{t \cos(2 t)}{4} \right\}$$

Problem 106. Solve the previous initial-value problem using the Laplace transformation.

Maxima:

```

(C106) ivp: diff(f(t), t, 2) + 4*f(t) = sin(2*t)$
(C107) atvalue(f(t), t = 0, 0)$
(C108) atvalue(diff(f(t), t), t = 0, 0)$ Lm : laplace( ivp, t, s );
(C109)
(D109) 
$$s^2 \text{LAPLACE}(f(t), t, s) + 4 \text{LAPLACE}(f(t), t, s) = \frac{2}{s^2 + 4}$$


(C110) sol : solve( Lm, 'laplace(f(t), t, s) );
(D110) 
$$[\text{LAPLACE}(f(t), t, s) = \frac{2}{s^2 + 8s + 16}]$$


(C111) ilt( sol[1], s, t );
(D111) 
$$f(t) = \frac{\text{SIN}(2t)}{8} - \frac{t \text{COS}(2t)}{4}$$


```

MuPAD:

```

>> de := f''(t) + 4*f(t) = sin(2*t): iv := f(0) = 0, f'(0) = 0:
>> Lm := transform::laplace( de, t, s )

4 transform::laplace(f(t), t, s) - D(f)(0) - s f(0) +


$$s^2 \text{transform::laplace}(f(t), t, s) = \frac{2}{s^2 + 4}$$


>> sol := op(subs(linsolve( [Lm],
&> [transform::laplace(f(t),t,s)] ), iv))


$$\text{transform::laplace}(f(t), t, s) = \frac{2}{8s^2 + s^4 + 16}$$


>> map( sol, transform::invlaplace, s, t )


$$f(t) = \frac{\text{sin}(2t)}{8} - \frac{t \text{cos}(2t)}{4}$$


```

Problem 107. Solve the differential equation  $x^2y'(x) + 3xy(x) = \sin x/x$ .

Maxima:

```
(C107) de : x^2*'diff(y, x) + 3*x*y = sin(x)/x$ ode2( de, y, x );
```

```
(C108)
```

```
(D108)          %C - COS(x)
           y = -----
                3
                x
```

MuPAD:

```
>> solve(ode( x^2*y'(x) + 3*x*y(x) = sin(x)/x, y(x) ))
```

$$\left\{ \begin{array}{l} C9 + \cos(x) \\ - \frac{\quad}{3} \\ \quad x \end{array} \right\}$$

Problem 108. Solve the differential equation  $y'' + y y'^3 = 0$  for  $y = y(x)$ .

Maxima:

```

(C108) ode2( 'diff(y, x, 2) + y*'diff(y, x)^3 = 0, y, x );
          3
          y  + 6 %K1 y
(D108)  ----- = x + %K2
          6

(C109) solve( %, y);
          Sqrt(3) %I  1
(D109) [y = (- ----- - -)
          2          2

          2          2          3          1/3
          (Sqrt(9 x  + 18 %K2 x + 9 %K2  + 8 %K1 ) + 3 x + 3 %K2)

          Sqrt(3) %I  1
          2 (----- - -) %K1
          2          2

          -----,
          2          2          3          1/3
          (Sqrt(9 x  + 18 %K2 x + 9 %K2  + 8 %K1 ) + 3 x + 3 %K2)

          Sqrt(3) %I  1
y = (- ----- - -)
          2          2

          2          2          3          1/3
          (Sqrt(9 x  + 18 %K2 x + 9 %K2  + 8 %K1 ) + 3 x + 3 %K2)

          Sqrt(3) %I  1
          2 (- ----- - -) %K1
          2          2

          -----,
          2          2          3          1/3
          (Sqrt(9 x  + 18 %K2 x + 9 %K2  + 8 %K1 ) + 3 x + 3 %K2)

          2          2          3          1/3
y = (Sqrt(9 x  + 18 %K2 x + 9 %K2  + 8 %K1 ) + 3 x + 3 %K2)

          2 %K1
          -----]
          2          2          3          1/3
          (Sqrt(9 x  + 18 %K2 x + 9 %K2  + 8 %K1 ) + 3 x + 3 %K2)

```

MuPAD:

```
>> solve(ode( y''(x) + y(x)*y'(x)^3 = 0, y(x) ));
```

$$\{C4, \text{RootOf}(X16^3 - 6 C5 - 6 C3 X16 - 6 x, X16)\}$$

```
>> solve( op(op(%,1)), MaxDegree = 3 )
```

$$\left\{ \frac{(3x^3 + 3C5 + (18x^2 C5 - 8C3^2 + 9x^2 + 9C5^{1/2})^{1/3})^{2/3} + \frac{2C3}{(3x^3 + 3C5 + (18x^2 C5 - 8C3^2 + 9x^2 + 9C5^{1/2})^{1/3})^{2/3}}}{(3x^3 + 3C5 + (18x^2 C5 - 8C3^2 + 9x^2 + 9C5^{1/2})^{1/3})^{1/2}} - \frac{1/2}{(3x^3 + 3C5 + (18x^2 C5 - 8C3^2 + 9x^2 + 9C5^{1/2})^{1/3})^{1/2}} \right\}$$

$$\begin{aligned}
 & \frac{1}{2} \int \frac{1}{3} \sqrt{\dots} \\
 & \frac{(3x^3 + 3C_5 + (18x^2 C_5 - 8C_3^2 + 9x^2 + 9C_5^2)^{1/2})^{1/3}}{(3x^3 + 3C_5 + (18x^2 C_5 - 8C_3^2 + 9x^2 + 9C_5^2)^{1/2})^{2/3}} - \frac{2C_3}{(3x^3 + 3C_5 + (18x^2 C_5 - 8C_3^2 + 9x^2 + 9C_5^2)^{1/2})^{1/3}} \\
 & \frac{C_3}{(3x^3 + 3C_5 + (18x^2 C_5 - 8C_3^2 + 9x^2 + 9C_5^2)^{1/2})^{1/3}} \\
 & \frac{(3x^3 + 3C_5 + (18x^2 C_5 - 8C_3^2 + 9x^2 + 9C_5^2)^{1/2})^{1/3}}{2}
 \end{aligned}$$

Problem 109. Solve the differential equation  $\frac{d y(a,x)}{dx} = a y(a,x)$ .

Maxima:

```

(C109) ode2( diff(y(x,a),x) = a*y(x,a), y(x,a), x );
(D109)          a x
              y(x, a) = %C %E
  
```

MuPAD:

```

>> solve(ode( diff(y(x,a),x) = a*y(x,a), y(x,a) ))
Error: Function must be univariate [ode::new]
  
```

Problem 110. Solve the boundary-value problem  $y''(x) + k^2 y(x) = 0, y(0) = y'(1) = 0$ .

Maxima:

```

(C110) de : 'diff(y, x, 2) + k^2*y = 0$ assume(not(equal(k,0)))$
(C111)
(C112) gsol : ode2( de, y, x );
(D112)      y = %K1 SIN(k x) + %K2 COS(k x)
(C113) bc1 : gsol, x = 0, y = 0;
(D113)      0 = %K2
(C114) der : diff(gsol, x)$ bc2 : der, 'diff(y,x)=0, x=1;
(C115)
(D115)      0 = %K1 k COS(k) - %K2 k SIN(k)
(C116) csol : solve([bc1, bc2], [%K1, %K2])$ ev(gsol, csol);
(C117)
(D117)      y = 0

```

MuPAD:

```

>> solve(ode( {y''(x) + k^2*y(x) = 0, y(0)=0, D(y)(1)=0}, y(x) ))
{0}

```

Problem 111. Solve the system of differential equations  $x'(t) = x(t) - y(t)$ ,  $y'(t) = x(t) + y(t)$ .

Maxima:

```

(C111) sde: [diff(x(t),t) = x(t)-y(t), diff(y(t),t) = x(t)+y(t)]$
(C112) sol: desolve( sde, [x(t), y(t)] );
          t (2 (- y(0) - x(0)) + 2 x(0)) SIN(t)
(D112) [x(t) = %E (-----
          2
          t (2 y(0) + 2 (x(0) - y(0))) SIN(t)
+ x(0) COS(t)), y(t) = %E (-----
          2
+ y(0) COS(t)]]

```

MuPAD:

```

>> sde := {diff(x(t),t)-x(t)+y(t) = 0,diff(y(t),t)-x(t)-y(t) = 0}:
>> sol := solve(ode( sde, {x(t), y(t)} ))
{[y(t) = C1 exp((1 + I) t) + C2 exp((1 - I) t),
 x(t) = I C1 exp((1 + I) t) - I C2 exp((1 - I) t)]}

```

Problem 112. Verify the solution of the previous problem.

Maxima:

```
(C112) ev( sde, ratsimp(sol), diff )$
(C113) ev( %, diff(x(0), t) = 0, diff(y(0), t) = 0 )$
(C114) map( lambda([z], is(equal(lhs(z), rhs(z))))), % );
(D114) [TRUE, TRUE]
```

MuPAD:

```
>> is(op(eval(subs( sde, op(sol) ))))
TRUE
```

Problem 113. Solve the system of differential equations  $x'(t) = x(t)(1 + \cos t/(2 + \sin(t)))$ ,  $y'(t) = x(t) - y(t)$ .

Maxima:

MuPAD:

```
>> sde := {x'(t) = x(t)*(1+cos(t)/(2+sin(t))),y'(t) = x(t)-y(t)}:
>> sol := simplify(op(solve( ode(sde, {x(t), y(t)}) )))

[y(t) = C7 exp(-t) + C3 exp(-t) int(exp(2 t)
(16 sin(t) - 2 cos(2 t) + 18)^(1/2), t),
x(t) = C3 exp(t) (16 sin(t) - 2 cos(2 t) + 18)^(1/2) ]
```

Problem 114. Solve the system of previous problem by solving one equation at a time.

Maxima:

```

(C114) sde : [diff(x(t),t) = x(t)*(1 + cos(t)/(2 + sin(t))),
diff(y(t),t) = x(t) - y(t)]$
(C115) ode2(sde[1], x(t), t)$ x_sol : subst(%C = %C1, factor(%));
(C116)

$$\frac{\text{SIN}(t)}{2 \text{ ATAN}\left(\frac{\text{SIN}(t)}{\text{COS}(t) + 1}\right)}$$

(D116) x(t) = %C1 (SIN(t) + 2) %E
(C117) ode2(subst(x_sol,sde[2]),y(t),t)$ y_sol: map('factorsum,%);
(C118)

$$\frac{\%E^{-t} (\%C1 \%E^{2t} (2 \text{ SIN}(t) - \text{COS}(t) + 5) + 5 \%C)}{5}$$

(D118) y(t) = -----

```

MuPAD:

```

>> sde := {x'(t) = x(t)*(1+cos(t)/(2+sin(t))),y'(t) = x(t)-y(t)}:
>> x_sol := solve( ode( sde[2], x(t) ) ) [1]

$$C15 \exp(t) (16 \sin(t) - 2 \cos(2 t) + 18)^{1/2}$$

>> y_sol := solve(ode( subs( sde[1], x(t) = x_sol ), y(t) )) [1]

$$C19 \exp(-t) + \frac{\int(C15 \exp(t) (16 \sin(t) - 2 \cos(2 t) + 18)^{1/2}, t)}{\exp(t)}$$


```

## 17 Operators

Problem 115. Define the operator  $L := (D - 1)(D + 2)$ , where  $D$  is the derivative operator.

Maxima:

```

(C115) L[f](x) := block([v], v: diff(f(x),x)+2*f(x), diff(v,x)-v);
(D115) L(x) := BLOCK([v], v : DIFF(f(x), x) + 2 f(x),
f
DIFF(v, x) - v)

```

MuPAD:

```
>> L := (D - id)@(D + 2*id)
(D - id)@(2 id + D)
```

Problem 116. Evaluate  $L(f)$ , where  $L$  is the operator defined in the previous problem.

Maxima:

```
(C116) L[f];
(D116) LAMBDA([x], --- (f(x)) + -- (f(x)) - 2 f(x))
          2          d
          dx
```

MuPAD:

```
>> L(f)
D(f) - 2 f + D(D(f))
```

Problem 117. Evaluate  $L(g)(y)$ , where  $L$  is the operator defined in the problem 115.

Maxima:

```
(C117) L[g](y);
(D117) --- (g(y)) + -- (g(y)) - 2 g(y)
          2          dy
```

MuPAD:

```
>> L(g)(y)
D(g)(y) - 2 g(y) + D(D(g))(y)
```

Problem 118. Apply the operator  $L$  to the function  $z \rightarrow A \sin z^2$ , where  $L$  is the operator defined in the problem 115.

Maxima:

```
(C118) h(z) := A*sin(z^2)$ L[h](z);
(C119)
(D119) - 4 A z2 SIN(z2) - 2 A SIN(z2) + 2 A z2 COS(z2)
+ 2 A COS(z2)
```

MuPAD:

```
>> L(z->A*sin(z^2))(z)
2 A cos(z2) - 2 A sin(z2) + 2 A z2 cos(z2) - 4 A z2 sin(z2)
```

Problem 119. Define the operator  $T$  so that  $T(f)(x,a) = f(a) + f'(a)(x - a) + f''(a)(x - a)^2/2$ .

Maxima:

```
(C119) T[f](x,a) := block( [u,v], v :
f(u) + diff(f(u), u)*(x - a) +
diff(f(u), u, 2)*(x - a)^2/2, at(v, u = a) );
(D119) T (x, a) := BLOCK([u, v],
f
v : f(u) + DIFF(f(u), u) (x - a) +  $\frac{\text{DIFF}(f(u), u, 2) (x - a)^2}{2}$ ,
AT(v, u = a))
```

MuPAD:

```
>> T := proc(f) option hold, escape; begin
&> context((x,a) -> f(a) + D(f)(a)*(x-a) + D(D(f))(a)*(x-a)^2/2)
&> end_proc;
```

Problem 120. Evaluate  $T(f)$ , where  $T$  is the operator defined in the previous problem.

Maxima:

(C120) T[f];

$$\frac{d^2}{du^2} (f(u)) \Big|_{u=a} (x-a)^2$$

(D120) LAMBDA([x, a], -----  
2

$$+ \frac{d}{du} (f(u)) \Big|_{u=a} (x-a) + f(a)$$

MuPAD:

>> T(f)

$$(x, a) \rightarrow f(a) + D(f)(a)*(x - a) + (D(D(f)))(a)*(x - a)^2/2$$

Problem 121. Evaluate  $T(g)(y,b)$ , where  $T$  is the operator defined in the problem 119.

Maxima:

(C121) T[g](y,b);

$$\frac{d^2}{du^2} (g(u)) \Big|_{u=b} (y-b)^2$$

(D121) ----- +

$$\frac{d}{du} (g(u)) \Big|_{u=b} (y-b)$$

$$+ g(b)$$

MuPAD:

```
>> T(g)(y,b)
```

$$g(b) + D(g)(b) (y - b) + \frac{D(D(g))(b) (y - b)^2}{2}$$

Problem 122. Apply the operator T to the function sin at (z,c), where T is the operator defined in the problem 119.

Maxima:

```
(C122) T[sin](z,c);
```

$$(D122) \quad - \frac{\text{SIN}(c) (z - c)^2}{2} + \text{COS}(c) (z - c) + \text{SIN}(c)$$

MuPAD:

```
>> eval( T(sin)(z,c) )
```

$$\text{sin}(c) + \text{cos}(c) (z - c) - \frac{\text{sin}(c) (z - c)^2}{2}$$

## 18 Programming

Problem 123. Define the Legendre polynomials.

Maxima:

```
(C123) p[n](x) := ratsimp( 1/(2^n*n!)*diff((x^2-1)^n, x, n) );
```

$$(D123) \quad p_n(x) := \text{RATSIMP}\left(\frac{1}{2^n n!} \text{DIFF}((x^2 - 1)^n, x, n)\right)$$

MuPAD:

```
>> p := (n,x) -> normal(1/(2^n*n!)*D([1 $ n],
```

```
&> u -> (u^2 - 1)^n)(x))
```

```
(n, x) -> normal((1/(2^n*fact(n)))*D([1 $ n], u->(u^2-1)^n)(x))
```

Problem 124. Define the Legendre polynomials recursively.

Maxima:

```
(C124) pp[0](x) := 1$ pp[1](x) := x$
(C125)
(C126) pp[n](x) := ratsimp( ((2*n-1)*x*pp[n-1](x) -
(n-1)*pp[n-2](x))/n );
(D126) pp (x) :=
      n
      (2 n - 1) x pp (x) - (n - 1) pp (x)
      n - 1 n - 2
RATSIMP(-----)
      n
```

MuPAD:

```
>> pp := proc(n,x) option remember; begin
&> if n = 0 then 1
&> elif n = 1 then x
&> else
&> normal( ((2*n - 1)*x*pp(n-1, x) - (n - 1)*pp(n-2, x))/n )
&> end_if
&> end_proc:
```

Problem 125. Evaluate the fourth Legendre polynomial at 1.

Maxima:

```
(C125) load(specfun)$ [p[4](1), pp[4](1), legendre_p(4,1)];
(C126)
(D126) [1, 1, 1]
```

MuPAD:

```
>> [p(4,1), pp(4,1), orthpoly::legendre(4,1)]
[1, 1, 1]
```

## 19 Translations

Problem 126. Define the polynomial  $p = \sum_{i=1}^5 a_i x^i$ .

Maxima:

```
(C126) p : sum( a[i]*x^i, i, 1, 5 );
(D126)      5      4      3      2
           a x + a x + a x + a x + a x
           5      4      3      2      1
```

MuPAD:

```
>> p := poly( sum(a[i]*x^i, i = 1..5), [x] )
           5      4      3      2
poly(a[5] x + a[4] x + a[3] x + a[2] x + a[1] x, [x])
```

Problem 127. Apply Horne's rule to the polynomial defined in the previous problem.

Maxima:

```
(C127) hp : horner( p );
(D127)      x ( x ( x ( x ( a x + a ) + a ) + a ) + a )
           5      4      3      2      1
```

MuPAD:

```
>> p(x)
           x ( a[1] + x ( a[2] + x ( a[3] + x ( a[4] + x a[5] ) ) ) )
```

Problem 128. Convert the result of previous problem to FORTRAN.

Maxima:

```
(C128) fortran( hp );
           x*(x*(x*(x*(a(5)*x+a(4))+a(3))+a(2))+a(1))
(D128)      DONE
```

MuPAD:

```
>> generate::fortran( p(x) )
           t5 = x*(a(1)+x*(a(2)+x*(a(3)+x*(a(4)+x*a(5))))
```

## 20 Logic

Problem 129. Simplify TRUE and FALSE.

Maxima:

```
(C129) true and false;
(D129)      FALSE
```

MuPAD:

```
>> TRUE and FALSE
```

```
FALSE
```

Problem 130. Simplify  $x$  or (not  $x$ ).

Maxima:

```
(C130) x or not x, x : true; x or not x, x : false;
```

```
(D130) TRUE
```

```
(C131)
```

```
(D131) TRUE
```

MuPAD:

```
>> x or not x
```

```
TRUE
```

Problem 131. Simplify  $x$  or  $y$  or ( $x$  and  $y$ ).

Maxima:

```
(C131) x or y or (x and y);
```

```
MACSYMA was unable to evaluate the predicate:
```

```
x
```

```
-- an error. Quitting. To debug this try DEBUGMODE(TRUE);)
```

MuPAD:

```
>> simplify( x or y or (x and y), logic )
```

```
x or y
```

## Summary

### Scores and times

In this summary the color coding of the solutions of the problems is changed to scores according to the following table.

	Colors and scores				
color	green	blue	black	red	magenta
score	3	2	1	0	-1

The following tables summarize the results of the previous sections. They contain also the calculation times (in seconds). In every table the last row indicates the maximal score, the sum of scores obtained by Maxima, the sum of scores obtained by MuPAD, the sum of times used by Maxima and the sum of times used by MuPAD. The color of the score shells of Maxima and MuPAD is chosen according to the following scheme; here  $n$  denotes the number of problems and  $s$  the score in the category:

Sum of scores and colors				
score	$-n \leq s \leq 0$	$0 < s \leq n$	$n < s \leq 2n$	$2n < s \leq 3n$
color	magenta	red	blue	green

[Numbers *after* topic headings in tables are links to the corresponding sections.]

## Numbers

Numbers 1				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
1	3	3	0.0	0.05
2	3	3	0.0	0.32
3	3	3	0.02	0.14
4	3	3	0.03	0.23
5	3	3	0.0	0.52
6	2	3	0.0	0.07
7	3	3	0.0	0.13
8	3	3	0.05	2.78
9	2	3	0.55	8.34
10	3	3	0.03	0.28
<b>30</b>	<b>28</b>	<b>30</b>	0.68	13.54

## Statistics

Statistics 2				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
11	2	3	0.47	0.0
12	0	3	-	0.48
13	0	3	-	0.48
<b>9</b>	<b>2</b>	<b>9</b>	0.47	1.43

## Algebra

Algebra 3				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
14	3	3	0.01	0.14
15	3	3	0.02	1.28
16	3	3	0.07	0.86
17	3	3	0.01	0.84
18	3	2	0.02	1.54
19	3	3	0.01	0.76
20	3	3	0.01	0.85
<b>21</b>	<b>21</b>	<b>20</b>	0.15	6.42

## Inequalities

Inequalities 4				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
21	-1	3	0.02	2.51
22	-1	3	0.01	1.61
23	0	3	-	3.15
24	0	3	-	2.68
<b>12</b>	<b>-2</b>	<b>12</b>	0.03	9.95

## Trigonometry

Trigonometry 5				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
25	3	3	0.06	0.29
26	3	3	0.13	0.32
27	3	3	0.05	0.12
<b>9</b>	<b>9</b>	<b>9</b>	0.24	0.73

## Finding zero

Finding zero 6				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
28	3	3	0.01	1.30
29	1	3	0.01	1.41
30	3	3	0.04	1.63
31	-1	0	6.14	-
32	3	2	0.02	2.73
33	3	3	0.22	1.87
34	2	2	0.07	3.07
<b>21</b>	<b>14</b>	<b>16</b>	6.51	12.01

## Complex domain

#	Complex domain 7			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
35	3	3	0.0	1.16
36	3	3	0.01	0.92
37	3	3	0.01	1.06
38	-1	3	0.02	2.66
39	-1	3	0.01	2.78
40	-1	3	0.0	0.36
41	-1	3	0.0	3.16
42	-1	3	0.0	0.36
43	-1	3	0.0	1.84
44	-1	3	0.0	1.58
45	-1	3	0.0	0.51
<b>33</b>	<b>1</b>	<b>33</b>	0.05	16.03

## Equations

#	Equations 8			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
46	3	3	0.0	0
47	2	2	2.78	4.66
48	3	3	0.06	1.5
49	3	3	0.11	0
50	1	3	0.04	4.89
51	1	3	0.04	2.23
52	1	3	0.01	1.74
53	0	3	0.02	2.73
54	1	3	0.03	1.83
55	1	3	0.02	2.23
56	0	3	0.01	1.29
57	3	3	0.03	4.52
58	0	3	0.02	3.25
59	0	3	0.02	3.09
60	3	3	0.01	0.25
61	2	2	11.76	4.36
<b>48</b>	<b>24</b>	<b>46</b>	14.96	38.57

## Matrix

#	Matrix 9			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
62	3	3	0.0	1.04
63	2	2	0.11	1.95
64	3	3	0.22	1.87
65	0	0	-	-
66	0	0	-	-
15	8	8	0.33	3.82

## Sums and products

#	Sums and products 10			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
67	3	3	0.03	1.04
68	3	3	0.01	3.07
69	2	3	0.21	3.04
70	3	3	0.01	2.03
12	11	12	0.23	8.14

## Limits

#	Limits 11			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
71	3	3	0.19	1.98
72	3	3	0.0	0.08
6	6	6	0.19	2.06

## Integration

#	Integration 12			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
73	3	2	0.1	6.87
74	2	2	0.06	99.33
75	3	1	0.61	11.36
76	3	3	0.0	0.68
77	0	3	0.01	1.32
78	0	3	0.01	0.24
79	0	3	0.0	0.45
80	0	3	0.23	5.09
81	2	3	0.02	1.32
27	13	23	0.94	126.66

## Definite integrals

Definite integrals 13				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
82	3	3	0.23	4.96
83	3	3	0.0	4.5
84	-1	1	0.23	0.13
85	3	1	0.25	0.48
86	-1	1	0.24	1.02
87	3	3	0.38	3.53
88	3	3	0.1	4.12
89	0	3	0.03	6.26
90	3	3	0.22	2.12
<b>27</b>	<b>16</b>	<b>21</b>	1.45	27.12

## Series

Series 14				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
91	2	3	0.02	0.69
92	2	3	0.02	0.19
93	3	3	0.02	0.66
94	2	3	0.03	1.53
95	2	3	0.0	1.38
96	-1	3	0.03	0.53
97	2	3	0.01	0.82
98	3	0	0.05	0.57
99	1	3	0.03	3.73
100	0	3	-	0.66
101	3	3	0.02	0.68
<b>33</b>	<b>19</b>	<b>30</b>	0.23	11.44

## Transforms

Transforms 15				
#	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
102	3	3	0.01	0.99
103	2	2	0.03	2.13
<b>6</b>	<b>5</b>	<b>5</b>	0.04	3.12

## Differential equations

#	Differential equations 16			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
104	3	3	0.46	1.51
105	3	3	0.0	1.73
106	3	3	0.04	1.65
107	3	3	0.05	10.32
108	2	2	0.12	3.72
109	3	0	0.02	-
110	1	1	0.05	2.96
111	3	3	0.0	2.82
112	2	3	0.0	0.11
113	0	2	-	73.48
114	3	2	0.82	34.99
<b>33</b>	<b>26</b>	<b>25</b>	1.56	133.29

## Operators

#	Operators 17			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
115	3	3	0.0	0.02
116	3	3	0.01	0.06
117	3	3	0.01	0.05
118	3	3	0.01	0.13
119	3	3	0.0	0.0
120	3	3	0.01	0
121	3	3	0.01	0.08
122	3	3	0.01	0.13
<b>24</b>	<b>24</b>	<b>24</b>	0.06	0.47

## Programming

#	Programming 18			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
123	3	3	0.0	0
124	3	3	0.0	0.0
125	3	3	0.59	0.1
<b>9</b>	<b>9</b>	<b>9</b>	0.59	0.1

## Translations

#	Translations 19			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
126	3	3	0.02	0.15
127	3	3	0.0	0
128	3	3	0.01	0.92
<b>9</b>	<b>9</b>	<b>9</b>	<b>0.03</b>	<b>1.07</b>

## Logic

#	Logic 20			
	Scores		Times	
	Maxima	MuPAD	Maxima	MuPAD
129	3	3	0.0	0
130	2	3	0.0	0
131	0	3	0.0	0.44
<b>9</b>	<b>5</b>	<b>9</b>	<b>0.0</b>	<b>0.44</b>

## Summary of scores

Topic	Summary of scores		
	Max	Maxima	MuPAD
Numbers 1	30	28	30
Statistics 2	9	2	9
Algebra 3	21	21	20
Inequalities 4	12	-2	12
Trigonometry 5	9	9	9
Finding zero 6	21	14	16
Complex domain 7	33	1	33
Equations 8	48	24	46
Matrix 9	15	8	8
Sums and products 10	12	11	12
Limits 11	6	6	6
Integration 12	27	13	23
Definite integrals 13	27	16	21
Series 14	33	19	30
Transforms 15	6	5	5
Differential equations 16	33	26	25
Operators 17	24	24	24
Programming 18	9	9	9
Translations 19	9	9	9
Logic 20	9	5	9
$\Sigma$	<b>393</b>	<b>247</b>	<b>356</b>

## Summary of times

Summary of times [seconds]		
Topic	Maxima	MuPAD
Numbers 1	0.68	13.54
Statistics 2	0.47	1.43
Algebra 3	0.15	6.42
Inequalities 4	0.03	9.95
Trigonometry 5	0.24	0.73
Finding zero 6	6.51	12.01
Complex domain 7	0.05	16.03
Equations 8	14.96	38.57
Matrix 9	0.33	3.82
Sums and products 10	0.23	8.14
Limits 11	0.19	2.06
Integration 12	0.94	126.66
Definite integrals 13	1.45	27.12
Series 14	0.23	11.44
Transforms 15	0.04	3.12
Differential equations 16	1.56	133.29
Operators 17	0.06	0.47
Programming 18	0.59	0.10
Translations 19	0.03	1.07
Logic 20	0.00	0.44
$\Sigma$	28.74	416.31

## Frequencies

The colors magenta, red, blue and green of the following table correspond the quartiles; in case of no or wrong answer the order of the colors is reversed.  
[Mama = Maxima, MuP = MuPAD.]

Topic	Frequencies of right, no and wrong solutions						
	Max	Right sol.		No sol.		Wrong sol.	
		Mama	MuP	Mama	MuP	Mama	MuP
Numbers 1	10	10	10	0	0	0	0
Statistics 2	3	1	3	2	0	0	0
Algebra 3	7	7	7	0	0	0	0
Inequalities 4	4	0	4	2	0	2	0
Trigonometry 5	3	3	3	0	0	0	0
Finding zero 6	7	6	6	0	1	1	0
Complex domain 7	11	3	11	0	0	8	0
Equations 8	16	12	16	4	0	0	0
Matrix 9	5	3	3	2	2	0	0
Sums and products 10	4	4	4	0	0	0	0
Limits 11	2	2	2	0	0	0	0
Integration 12	9	5	9	4	0	0	0
Definite integrals 13	9	6	9	1	0	2	0
Series 14	11	9	10	1	1	1	0
Transforms 15	2	2	2	0	0	0	0
Differential equations 16	11	10	10	1	1	0	0
Operators 17	8	8	8	0	0	0	0
Programming 18	3	3	3	0	0	0	0
Translations 19	3	3	3	0	0	0	0
Logic 20	3	2	3	1	0	0	0
$\Sigma$	131	99	126	18	5	14	0

## Conclusions

Maxima obtained better scores than MuPAD in two topics: "Algebra" and "Differential equations". MuPAD was better than Maxima in 11 topics. The scores were equal in 7 topics. Out of the 20 different topics Maxima got maximum scores in 6 topics and MuPAD got those in 11 topics. The percentages of the maximum score were 63% for Maxima and 91% for MuPAD.

The percentages of (partially or fully) *solved problems* were 76% for Maxima and 96% for MuPAD. The percentages of *non-solved problems* were 14% for Maxima and 4% for MuPAD. Maxima gave a *wrong solution* to 11% of the problems whereas MuPAD gave no wrong solutions.

The total computation time used by Maxima was only 29 seconds whereas MuPAD used almost 7 minutes.

In light of the previous figures and tables the choice between Maxima and MuPAD is easy: MuPAD is the clear winner. However, if (your) time is (your) money and you know how to get around partial or wrong answers, Maxima is a good alternative.

## Colophon

This document was prepared using Java version of Skribe text processor; see <http://www-sop.inria.fr/mimosa/fp/Skribe>. The Skribe-source file was written with iTeXMac; see <http://itexmac.sourceforge.net>.

The input and output expressions of Maxima and MuPAD were copied *unmodified* into the source document from the terminal running the corresponding program. Both html- and pdf-documents were produced from this source file using Skribe's html- and latex-engines.

The calculation times were recorded using Maxima-declaration `showtime : all` and MuPAD-function `time()`. The computer was iMac DV, which has 400 MHz PowerPC G3-processor and 320 Mt SDRAM, running MacOS X version 10.3.2. Following programs and systems have been used during preparation of this document. Free programs are marked with blue color and the programs, which are part of Mac OS X, are green.

- [AppleScript](#) [1.9.3]
- [gwtex](#) [2003/12/04]
- [iTeXMac](#) [1.3.15]
- [Java](#) [1.4.2\_03]
- [Maxima](#) [5.9.0]
- [MuPAD](#) [2.5.0]
- [Preview](#) [2.1.0]
- [Safari](#) [1.2]
- [Script Editor](#) [2.0]
- [Skribe](#) [1.0b]
- [Smile](#) [2.5.9]
- [TextEdit](#) [1.3]
- [Terminal](#) [1.4.1]
- Many [UNIX](#) utilities

All these were running smoothly under Mac OS X in an iMac DV. It's good to have a Mac!

The icon below is a modification from a drawing by H. HÖGMANDER used by his permission.

## Version history

### Version 1.0

- First public release

### Version 2.0

- Added calculation times in the summary tables
- Added some comments on times in Conclusions
- Added a description of the timing system in Colophon
- Added program version numbers in Colophon

### Version 2.1

- [Thanks to STEFAN WEHMEIER] Corrected the MuPAD-solution to the problem 88. The line  
`assume(a > 0): assume(a < 1):`  
changed to  
`assume(a > 0): assume(a < 1, _and):`  
Consequently the problem 88 is completely solved by MuPAD instead of no solution.
- Made necessary changes to Summary

Version 2.1

Mon Mar 15 15:33:31 EET 2004

